

Relationship between focal length and magnification

Or how to evaluate the magnification from the focal length and vice versa

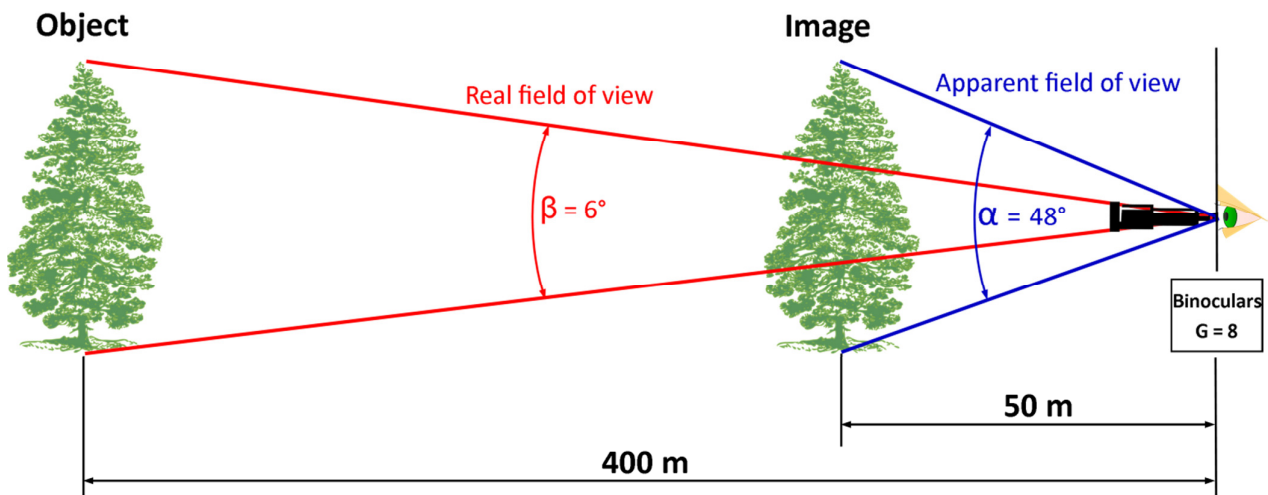
In this post, we will try to answer a question that I have often been asked. "Tell me, your super telephoto lens, how much does it magnify ?" or "What focal length you reach with your scope ? " Let's start by explaining the different labels found on the binoculars and field spotting scopes we use for the purpose of bird-watching for example. We will then try to assess the relationship between magnification of binoculars or scopes and the focal length of a camera lens. We finally will see what useful information we can learn out of it to choose and make the best use of these tools.

First of all, let's be clear, binoculars or telescopes do not possess a focal length nor a diaphragm, unlike a camera lens. Similarly, a camera lens is never characterized by a magnification but by a focal length and a maximum opening of its diaphragm (for example " *Canon EF 300 mm 1:2.8 L IS USM* " where 300 mm is the focal length and 1:2.8 is the maximum aperture of its diaphragm).

Magnification.

The magnification is defined as the ratio of the angular size of the image to the angular size of the object.

So magnification $G = \frac{\alpha}{\beta}$ with α = angular size of image
 β = angular size of object



According to the diagram above, it can also be admitted that an object will appear to be 8 times closer to the observer using binoculars of $G = 8$. This is a more commonly accepted definition and better understood by most people.

Now, let's see if we can make a connection between the focal length of a lens and its equivalent magnification. It will be easier to understand by most people, the focal length being often only an abstract data.

It is accepted that the human eye has a magnification of 1. It is also generally accepted that a lens with a focal length of 50 mm provides a very close field of view of the vision of our eye, hence having also a magnification of 1. In fact, and for the purists, this focal length is 43 mm. We'll just use the very good approximation of 50 mm for the rest of our words. From there, we can say that the magnification of a lens is equal to the focal length divided by 50.



So : $G_{lens} = \frac{F}{50}$ with F = focal length of the lens

Therefore, a super telephoto lens that shows **500 mm 1:4** provides a magnification of 10x (ie $500/50 = 10$) and its luminosity is represented by the number 4 (also indicated by $f/4$). This figure has no dimension. It actually represents the maximum possible diameter of the diaphragm, the maximum amount of light that reaches the film or digital sensor. This diameter may be calculated using the following formula

$$\Phi_{aperture} = \frac{F}{f} \quad \text{with } F = \text{focal length}$$

$f = \text{aperture number}$

$\Phi_{aperture}$ can also be considered as the entry pupil of the lens

For our telephoto lens at full aperture, diaphragm has a diameter of 125 mm ($f/4$) and 15.6 mm at its smallest aperture ($f/32$). Note that the entry pupil at maximum aperture is also the diameter of the front lens. We see that at constant focal length, if the number of aperture increases the diameter of opening decreases. So the bigger the aperture number you select on your device, the smaller the "hole" through which light will pass to reach the digital sensor. This translates into a loss of light which you can compensate by a slower shutter speed or by an increase of the sensitivity of the sensor (the famous ISO).

The binoculars

What piece of information can we find on a pair of binoculars ?



First indication : **10 x 42**

The first number is the magnification of the binoculars and the second gives the diameter of the front lens. From what we said above, we can say that the equivalent focal length of these binoculars is

$$F = G \times 50 = 10 \times 50 = 500 \text{ mm}$$

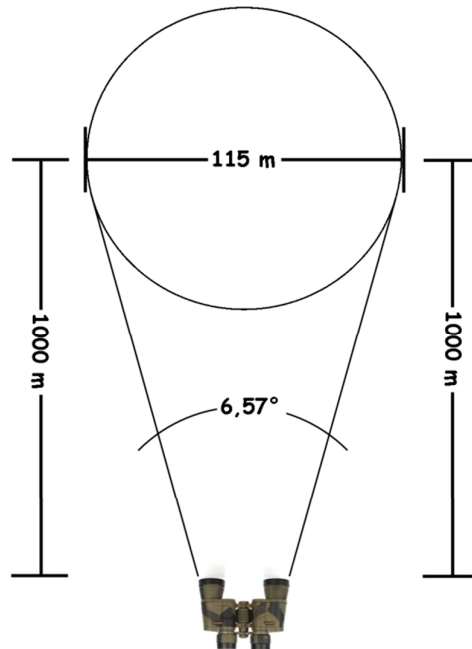
However let's not forget that binoculars have no focal length and that this conclusion is only an indication for the purpose of possible comparison with a camera lens.

The second number gives us the diameter of the front lens. This is a very good indication of the brightness of the binoculars. Indeed, the bigger the lens is, the more light will get through it to your eye. So a high number gives you high brightness.



Second indication : **FOV 341 FT or 346 FT @ 1000 YDS**
115 M @ 1000 M

The real field of view (FOV) is the angular dimension of the object seen from the center of the binoculars. It is given by the manufacturer in degrees (e.g. 6.6 °). Manufacturers express it more often in a number of meters at 1000 meters or a number of feet at 1000 yards which is easier to understand. The smaller the magnification is, the bigger the real field of view is, and conversely, a stronger magnification gives a narrower field of view. Therefore, the real fields of view of different magnification binoculars are not comparable.



One needs to recognize that it is not always easy to convert feet to meters, yards to meters and even less a number of feet at 1000 yards to a number of meters at 1000 meters. It is however easy to calculate that 1 foot @ 1000 yards is 0.333 m @ 1000 m and 1 m @ 1000 m is equal to 3,003 feet @ 1000 yards. So 346 feet @ 1000 yards = (346 x 0.333) m @ 1000 m = 115.2 m @ 1000 m.

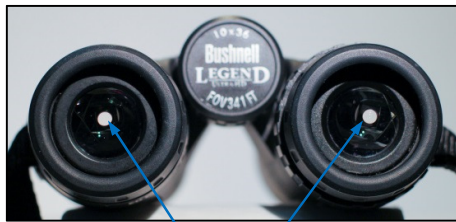
The field of view in degrees is a lot less often mentioned on the binoculars themselves. You will have to consult the technical documentation to find it. Again this expression of the field of view in degrees is more difficult to imagine. Let us make a little trigonometric digress. We know that a milliradian is an angle that intercepts an arc of 1 mm to 1 meter, 1 meter to 1000 meters (oh oh...) etc...

$$\begin{aligned} 360^\circ &= 2 \pi \text{ rad} \\ 1^\circ &= \frac{2 \pi \text{ rad}}{360} = \frac{\pi}{180} \text{ rad} \\ &= 0,01745 \text{ rad} \\ &= 17,45 \text{ milliradians} \end{aligned}$$

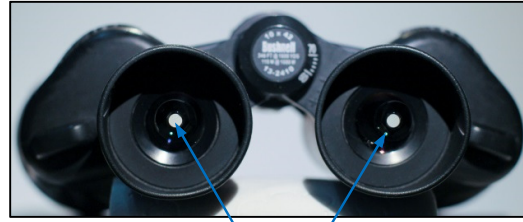
So, a degree sustains an arc of 17,45 meters at a distance of 1000 meters, and a field of view of 115 meters at 1000 meters equals 115/17.45 = 6.59 degrees. Similarly, 6.59 degrees represent a field of view of 17.45 x 6.59 = 115 meters to 1000 meters.

The exit pupil

It actually is the round point of light observed when one holds the binoculars with full extended arms, representing the diameter in millimeters of the image of the front lens shown through the eyepiece. It is defined by the ratio of the diameter of the front lens divided by the magnification. For example, a pair of binoculars 10 x 42 would have an exit pupil of $42/10 = 4.2$ mm.

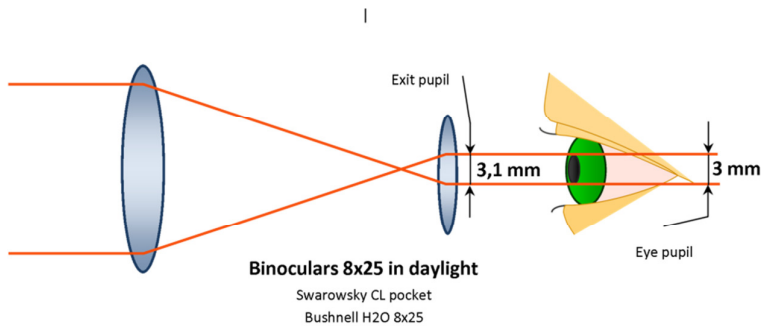


Exit pupil

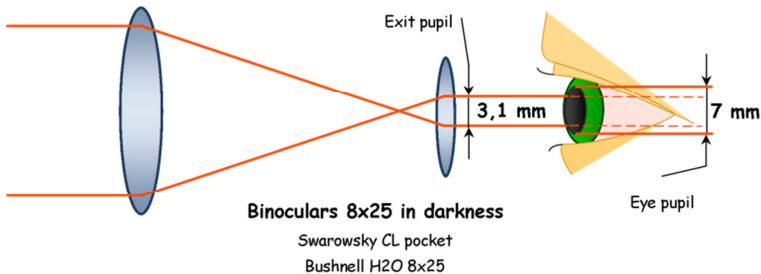


Exit pupil

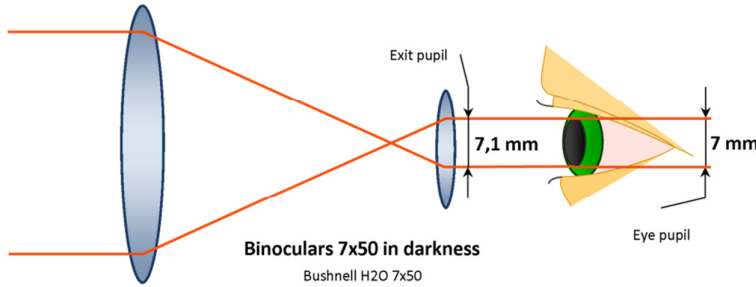
Ideally this diameter must be equal to that of the pupil of the eye for optimal use of light. If the pupil of the eye is different from the exit pupil, there is a more or less significant loss of brightness. In the dark, the pupils dilate in order to let more light into the eye ball. Conversely, in daylight, they contract in order to limit the amount of light reaching the retina to avoid glare. This expansion or contraction of the pupils becomes more difficult with age, the pupil being less flexible. The diameter of our pupil measures in the middle of the day between 2 and 3 mm, at dusk between 4 and 6 mm and at night around 7 mm.



The pupil of the human eye opening to 2 to 3 mm maximum in broad daylight, the exit pupil of the binoculars should measure about 3 mm.



At night, on the other hand, our pupils dilate up to 7 mm in the dark, an exit pupil of about 3 mm will not let enough light to get through, which will make the image dark.



Binoculars for night use must therefore have large exit pupils which makes them unfortunately heavier and so much more cumbersome.

We can check what has been said in supporting the technical data provided by Swarovsky for two of its pairs of binoculars

	EL 8x32 W B	EL 10x32 W B
Magnification	8x	10x
Lens diameter	32	32
Exit pupil diameter	4	3.2
Eye-to-eyepiece distance	20	20
Field of view in meters	141	120
Field of view in degrees	8	6.9

Twilight factor

This figure shows the optical quality of binoculars in low light. It is obtained by extracting the square root of the product of the diameter of the front lens with magnification. For example, a pair of binoculars 10 x 42 have a twilight factor of $\sqrt{(10 \times 42)} = \sqrt{420} = 20.49$. The binoculars that best suit night observation are the ones whose twilight factor equals or exceeds 49. However, we must relativize this result because it tends to favor big diameters. In addition it is virtually impossible to reach with a pair of binoculars. It could be reached with a model of 20 x 120 for example!

The following table gives exit pupils and twilight factors of the most common binoculars from two well-known brands.

	Swarowsky					
	8 x 25	8 x 30	8 x 32	8 x 42	8 x 56	10 x 50
Exit pupil	3,1	3,8	4,0	5,3	7,0	5,0
Twilight factor	14,1	15,5	16,0	18,3	21,2	22,4

	Bushnell							
	7 x 26	16 x 32	10 x 36	10 x 42	7 x 50	12 x 50	16 x 50	20 x 50
Exit pupil	3,7	2,0	3,6	4,2	7,1	4,2	3,1	2,5
Twilight factor	13,5	22,6	19,0	20,5	18,7	24,5	28,3	31,6

The scope

What has been said previously regarding the binoculars remains valid and of application for a scope. Let's take the Swarovski ATS/STS 80 HD technical data sheet.



Eyepiece	ATS/STS 80 HD	
	25-50x	20-60x
Magnification	25-50x	20-60x
Lens diameter	80	80
Exit pupil diameter (mm)	3.2-1.6	4.0-1.3
Field of view (m/1000 m)	42-27	36-20
Real field of view (degrees)	2.4-1.55	2.1-1.1
Apparent field of view (degrees)	60-70	40-65
Focal length with TLS/APO (mm)	750-1500	600-1800

Let's take the example of the scope equipped with a 25-50 x eyepiece.

1. Magnification.

It is the first number characterizing the binoculars (**10x 42**). We have here two numbers because it is a magnifying zoom of 25x to 50x.

2. Effective lens diameter.

This is the diameter of the front lens as for the binoculars. It is the second number (**10 x42**). We can consider that it also represents the size of the pupil to entry of the scope.

3. Diameter of the exit pupil.

We use the same formula to calculate its value.

$$\text{Exit pupil} = \frac{\phi}{G} = \frac{80}{25} = 3.2 \quad \text{and} \quad \frac{80}{50} = 1.6$$

4. Field of view and real field of view.

Given and probably measured by the manufacturer, we can check them out using the following conversion: 1 ° = 17,45 meters to 1000 meters.

2,4° corresponds to 2.4 x 17.45 = 41.88 m @ 1000 m

1.55 ° corresponds to 1.55 x 17.45 = 27.04 m @ 100 m

5. Apparent field of view.

It represents the angle under which the picture is seen through the eyepiece.

It can be calculated by G x real field of view. So 2.4° x 25 = 60 °

6. The problem of the focal length.

As mentioned above, the focal length is equal to the magnification x 50 (G x 50). Therefore it is reasonable to consider an equivalent focal length for this scope between 1250 mm and 2500 mm. For Digi scoping, we need to add to this scope a DSLR camera for example, equipped with its own lens. This lens has of course its own magnification calculated by the formula G = focal length/magnification. In the case of the TLS/APO, its focal length being 30 mm, its magnification is 0.6. The one of the combination scope + TLS/APO thus varies between 0.6 x 25 = 15 and 0.6 x 50 = 30. Now we can recalculate the entire focal length and we find 750 mm at 25 x and 1500 mm at 50x.